Understanding secondary school students' motivations for mathematics subject choice: First steps in construct validation and correlational analysis

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With the increased workplace demand for STEM specialists, and the trend in capable students opting out of higher levels of secondary mathematics, the psychological influences on mathematics subject choice are important issues to explore. Expectancy-value theory is used to examine the factors influencing such achievement choices. In the present study, as part of a larger programme of research on mathematics subject choice, we sought to validate self-report measures of students' expectancies for success, values, and perceived costs associated with participation in mathematics. Confirmatory factor analysis supported the hypothesised factor structure, with the measures displaying acceptable levels of internal consistency.

The growing demand for specialist STEM practitioners is undercut by a decline in participation in sciences and advanced mathematics in school and university (Australian Government Department of Education and Training, 2016), and an associated labour shortage in these fields. In the final two years of secondary education, a trend exists in which talented and capable students are turning away from the more rigorous calculus-based mathematics courses. In the state of New South Wales, Australia, there has been a decline from 34 to 22 percent of students opting for these higher-level courses over the past two decades (Jaremus et al., 2018). The calculus-based courses of study available to Australian students are Advanced Mathematics (previously known as Mathematics), Extension 1 and Extension 2 mathematics, and these courses lay the groundwork for meeting the challenges of many tertiary STEM pathways. Not completing adequate levels of high school mathematics preparation is associated with attrition from undergraduate STEM majors, with students being almost twice as likely to fail certain first-year science units if they did not complete a calculus-based mathematics course in secondary school (Nicholas et al., 2015).

It is important, therefore, to explore the antecedents of students making choices either for or away from participation in higher levels of upper secondary mathematics. Why are students, especially girls, increasingly dissuaded from choosing calculus level mathematics? What are the psychological influences on this choice? If we had a better understanding of these factors, might we be able to increase participation in these courses through levers in the middle school experience and the curriculum?

The current research investigates motivations for pursuing mathematics subjects in senior secondary school, with a focus on examining gender differences in motivational influences. The *Expectancy-Value Theory* (EVT) is drawn on as the guiding theoretical framework (Eccles et al., 1983; Wigfield & Eccles, 2000), being one of the most comprehensive frameworks for studying the psychological and contextual factors influencing individual and gender differences in achievement choices (Wigfield & Eccles, 2000). This theory has been used extensively to examine the short-term and long-term motivations and achievement outcomes in a variety of achievement domains.

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Research into student motivations using the EVT has predominantly been quantitative longitudinal variable-centred studies tracking changes in positive motivations throughout school, and their contributions to achievement-behaviour. These positive aspects of motivations are broken into the student's expectancy for success (perceived competence or self-concept) and their valuing of mathematics in the forms of utility value (perceived usefulness), attainment value (importance of doing well) and intrinsic value (inherent interest, a similar concept to intrinsic motivation). Historically, few studies have incorporated the negative "cost" component of the theory into empirical analyses. Cost refers to the perceived drawbacks of engaging in an activity and has been defined as the negative consequences derived from participating in an activity, such as perceived difficulties, fear of failure and loss of valued alternative activities (Wigfield, 1994). In more recent years, a fast-growing literature has begun to focus on measuring cost as a multidimensional construct (e.g., Barron & Hulleman, 2015; Battle & Wigfield, 2003; Chen & Liu, 2009; Chiang et al., 2011; Conley, 2012; Flake et al., 2015; Perez, et al., 2014; Watkinson et al., 2005).

The ongoing study extends this work by focusing on the influence of this negative cost factor in relation to the motivation of high school mathematics students, to further explore its associated effects on mathematics-related academic choices. The results of 500 survey responses collected from Year 10 students in New South Wales are analysed to derive motivational profiles of students with similar beliefs in their levels of expectancy for success, the values they hold for mathematics, and the costs they associate with this subject. A latent profile analysis will be conducted to identify and classify clusters of individuals with similar beliefs based on patterns of categorical responses, followed by interviews of students with high-cost profiles in an attempt to capture a further understanding of their experiences, the complexity of the interrelated influences on their motivation, and their influences on their choice in mathematics studies.

This paper presents initial results of the quantitative analyses assessing the construct validity and reliability of hypothesised constructs. It provides the groundwork for subsequent intended research exploring gendered relationships between and among the various motivational profiles, as well as their relationship to achievement background, language background, dependency on selective schools, coeducational/single-sex learning environments, amongst other educational contexts. These further factors will be explored through both quantitative and qualitative research components to follow. Results from this study can help provide information for school-teachers in understanding factors affecting student motivation and the types of classroom experiences and programs that may help shift students into more favourable motivational profiles, so students may be more likely to persist with a level of mathematics commensurate with their ability.

Method

Participants

Survey data were collected from 521 Year 10 students from 10 high schools in the Sydney metropolitan area. Data were gathered from participants at a critical decision point in relation to subject choice: students completed the surveys after having submitted their subject selection forms, and so were able to report their chosen level of mathematics for Year 11. All students in the Year 10 cohort were invited to participate and were given paper consent forms to be signed by parents or guardians and themselves. Their teachers were asked to remind students to return forms to maximise returns from each school.

New South Wales secondary school contexts vary significantly in demographic characteristics, numeracy performance, and level of participation in senior mathematics. The current study's sample includes a range of coeducational and single sex schools, comprehensive and selective schools. Some of this complexity was reduced by only including government schools (none from the Catholic or private sector), and by employing strategic sampling of schools. Participating schools were matched for socio-economic status to minimise its influence as a confounding variable, measured by the Index of Community Socio-Educational Advantage (ICSEA). This index is calculated based on student-level data on a raft of factors including family background, parental level of education, and remoteness of the school (Australian Curriculum and Assessment and Reporting Authority, 2018). The participating schools' mean ICSEA was 1082 (*SD* = 78.33), above the sector-wide mean ICSEA value of 1000 (*SD* = 100). Three schools were academically selective, seven were coeducational, two were girls-only and one was boys-only.

Instrument

The survey instrument first collected information on school attributes, including subject preferences and academic aspirations. The questions that followed gathered information on the level of mathematics the students had chosen for their final two years of high school and how they believed that it matched with their ability level ("Was this level of mathematics higher than/the same as/lower than what you believe you're capable of?"). There were also three open-ended short-answer questions eliciting students' reasons for their choice in level of mathematics. Follow-up interviews in the second qualitative part of this study will further clarify student responses to these questions.

This section was followed by 31 items gathering students' perceived expectancy and value (utility, attainment, intrinsic) beliefs, which were sourced from Eccles' Expectancy Value measures (Eccles, 2005; Eccles & Wigfield, 1995), with grammatical and contextualising modifications for the Australian sample developed and psychometrically validated in Australia (see Watt, 2004). Examples of some items are: "How well do you expect to do in your next maths task?" to measure expectancy for success or self-efficacy, "How useful do you think maths is in the everyday world?" to tap on utility value, "Being someone who is good at maths is important to me" to tap on attainment value, and "How enjoyable do you find maths?" to tap on intrinsic value.

The items measuring the dimensions of cost, including effort cost, outside effort cost, loss of valued alternatives cost, and emotional cost, were based on Flake et al.'s (2015) comprehensive scale validation study, with "this class" replaced by "mathematics". Examples of items were "I worry too much about mathematics" to tap on emotional cost, "Mathematics requires me to give up too many other activities I value" to tap on loss of valued alternatives, "Because of the all the other demands on my time, I don't have enough time for mathematics" to tap on outside effort cost, and "Mathematics demands too much of my time" to tap on task effort cost. Each expectancy, value, and cost item was rated on a 7-point Likert scale from 1 (not at all) to 7 (extremely). A question at the end of the questionnaire elicited student interest in participating in a short, individual, semi-structured interview early in the following year to further explore quantitative results and how subject choice are shaped by the various interrelated and interacting facets of motivation. For a copy of the full survey please contact the first author via email.

Procedure

Surveys were conducted in class, online via the Qualtrics survey platform, and were led by the students' normal classroom teacher. Respondents (*N* = 21) who provided insincere responses (e.g. pattern drawing, string responses) were excluded from the analyses. Missing data were rare as the online format of the survey ensured that important questions could not be skipped; however, respondents who exited the survey without completing it were also excluded from the analyses. The final sample consisted of 500 students (239 boys, 250 girls, 11 other, mean age = 15.69, SD = 0.77).

Confirmatory factor analysis (CFA) was used to assess the dimensionality of latent constructs using M*plus* 6.12 (Muthén and Muthén 2004). Multivariate normality is a key assumption of a range of multivariate statistical methods, including CFA (Kline, 2016). Mardia's (1970) test indicated the data were multivariate non-normal. To account for this, robust maximum likelihood estimation of covariance matrices was used, as this procedure is less sensitive than other estimation methods to violations of the normality assumption (Boomsma & Hoogland, 2001). Each of the latent motivation constructs of expectancies, values and costs were analysed for fit, with their corresponding 3 to 6 items as indicators for their assigned latent constructs.

To assess the reliability of survey measures, McDonald's (1999) omega was used as an estimate of internal consistency. There has been increasing criticism of the use of Cronbach's alpha in behavioural science research due to some of its untenable assumptions. Some of these assumptions include the requirement that each indicator variable contributes equally to the factor (tau-equivalence), and that error variances must be uncorrelated (Dunn et al., 2013). McDonald's omega takes into account the strength of association between items, as Cronbach's alpha's failure to do so may overestimate the reliability of results (Dunn et al., 2013). These initial procedures will ensure the consistency, validity and reliability of the latent constructs measured for the purposes of this study.

Results and Discussion

Confirmatory factor analysis confirmed that the eight-factor model of motivation to be a good fit to the data. Model fit was evaluated using recommendations by Kline (2016) and Marsh et al. (2004), focusing on the Comparative Fit Index (CFI), Tucker-Lewis Index (TLI), the root mean squared error of approximation (RMSEA), and the standardised root mean squared residual (SRMR). By these recommendations, RMSEA values at less than 0.08 are considered acceptable fit and values less than 0.05 are considered excellent fit (Marsh et al., 1996). For the CFI, values at or greater than 0.95 are taken to reflect excellent fit to the data (McDonald & Marsh, 1990). Cut-off values close to 0.95 for TLI; close to 0.08 for SRMR (Hu & Bentler, 1999) are considered acceptable fit.

The present eight-factor model showed an acceptable fit for each of the constructs (χ^2 = 788.76, df = 437, CFI = 0.964, TLI = 0.959, RMSEA = 0.040, SRMR = 0.042). Table 1 presents factor loading ranges of items against the hypothesised latent constructs, as well as descriptive statistics, reliability (using estimates of McDonald's omega). Where single item indicators (e.g. Gender, NESB, Co-educational/Single-sex school, Comprehensive/selective school, NAPLAN achievement) were used, the variance of these indicators was fixed at one, and standard deviation fixed at zero. For each sub-dimension, estimates of reliability using McDonald's omega ranged from 0.87 to 0.93, which indicates a high degree of internal consistency for all scales; factor loadings were strong (> .60), indicating items were suitably measuring hypothesised constructs. Descriptive statistics, McDonald's omega measures of reliability are also provided in Table 1.

Table 1
Descriptive statistics, reliability using McDonald's omega, confirmatory factor analysis factor loadings and measurement errors for each subconstruct

	Mean	SD	Skewness	Kurtosis	McDonald's ω	CFA Factor Loading Range	Mean Residuals
Expectancies	4.91	1.32	– 0.77	0.57	0.93	0.88 – 0.92	0.01
Intrinsic value	4.39	1.63	– 0.56	– 0.19	0.93	0.89 – 0.94	0.01
Attainment value	4.62	1.55	– 0.42	– 0.66	0.87	0.83 – 0.84	0.02
Utility value	5.02	1.37	– 0.45	– 0.53	0.89	0.84 – 0.89	0.02
Task effort cost	4.08	1.36	– 0.01	– 0.40	0.89	0.63 – 0.82	0.02
Outside effort cost	3.83	1.43	0.20	– 0.47	0.92	0.81 – 0.88	0.02
Loss of valued alternatives	3.48	1.41	0.22	– 0.23	0.88	0.74 – 0.85	0.02
Emotional cost	4.17	1.52	– 0.06	– 0.70	0.92	0.60 – 0.88	0.02

A correlational analysis showed that each measure of cost was negatively correlated to each of the positive subconstructs of motivation, which was to be expected. Some cost subscales were found to be highly correlated with one another, for instance, the correlation between task effort cost and emotional cost being 0.87. This level of correlation is not ideal, as a correlation of 1 means that the constructs are indistinguishable. The scale development study from which the current survey is based (Flake et al., 2015) found similar correlations in their initial analyses into measuring and operationalising the "cost" component for motivation. Their confirmatory factor analyses provided the strongest support for the four-factor solution of cost.

However, Flake et al. (2015) also found that the higher order factor model, which included a general unidimensional cost factor, also provided a good fit to the data. They argued that although the four-factor solution provided four highly-correlated dimensions, it showed adequate reliability and model fit, and a further correlational study revealed that the four cost factors had different relationships to the other positive motivation factors. This particular scale development study was conducted in a tertiary calculus setting with a smaller cohort (N = 228), which may explain the discrepancy between those results and the ones produced in the current secondary setting. Flake et al. suggested that future research should investigate the empirical structure of cost within different groups of students in different contexts to see how their findings might replicate across educational settings. The current study provided one such further context and showed that the cost factors also displayed high levels of multi-collinearity. Table 2 presents a latent correlation matrix for the constructs under analysis.

Table 2
Latent factor correlations for Expectancy, Values and Costs perceptions

	EXP	IV	AV	UV	TEC	OEC	LOVA	EMC
EXP	–							
IV	0.66	–						
AV	0.65	0.80	–					
UV	0.37	0.57	0.68	–				
TEC	– 0.39	– 0.55	– 0.42	– 0.35	–			
OEC	– 0.41	– 0.46	– 0.37	– 0.28	0.80	–		
LOVA	– 0.35	– 0.43	– 0.32	– 0.29	0.86	0.82	–	
EMC	– 0.51	– 0.63	– 0.46	– 0.31	0.87	0.72	0.73	–

Note. All correlation coefficients are statistically significant at the $p < 0.001$ level. EXP = expectancy for success, IV = intrinsic value, AV = attainment value, UV = utility value, TEC = task effort cost, OEC = outside effort cost, LOVA = loss of valued alternatives, EMC = emotional cost.

Correlations between the expectancies and values dimensions echoed that of comparable previous studies of secondary students' mathematics motivations using EVT (e.g. Watt, 2004). The highest correlation between these positive factors were between intrinsic value and attainment value ($r = 0.80$), followed by correlations between utility value and attainment value ($r = 0.68$) and between expectancy for success and intrinsic value ($r = 0.66$). In the current sample, although the correlations between the cost constructs were found to be high, the model also had a good level of fit and the cost sub-constructs were found to be differentially related to expectancies for success and values. Interestingly, emotional cost was related to intrinsic value more than any other cost component, which brings up the question of how emotional cost and the psychological cost of failure impacts on high school students' intrinsic valuing for mathematics. This question, along with others, will be further explored in the subsequent interview study with a subset of the survey participants.

Conclusion

The factorial structure of the underlying constructs was validated using CFA, with the measurement model confirmed to be valid and ready to be used for further analyses on relationships between the latent variables. High degrees of internal consistency showed that the items were reliable in measuring the constructs they were designed to measure. The fit indices were adequate, which was expected because the expectancy and value scales have undergone rigorous scale validation through multiple studies, across many year groups and in a variety of subject contexts. However, the cost scales displayed a level of multi-collinearity, and were problematic in some pairs of sub-constructs having a higher level of correlation. As the particular scale development study was conducted in a tertiary calculus setting with a smaller cohort, further work needs to be done examining the construct and dimensionality of the cost factor in the secondary context.

The present study provides a foundation for subsequent intended research linking students' mathematics motivational profiles with their school contexts and choice of mathematics course. The next steps in analyses include conducting a latent profile analysis to explore how students hold multiple motivational beliefs simultaneously to make decisions on persisting with difficult mathematics subjects, rather than examining the isolated effects of single variables. Previous work on motivational profiles have shown different profiles to be differentially related to persistence outcomes (Perez et al., 2014; Watt et al., 2019). Studies of cost have repeatedly shown that the theorised dimensions of cost contribute differentially to student motivations and have suggested that future research should seek to understand the sources of cost.

Without an understanding of how costs interact with the other expectancy and value components, and by excluding it from the EVT framework, research findings about motivational influences may be compromised. An imbalanced value-cost relationship may hinder motivation, so the planned interviews will seek to understand the experiences of students to gather the reasons and sources for the costs they perceive. “What could teachers do to optimise student motivation if they knew students were experiencing high cost?” was a question that Flake et al. (2015) posed in their study, and highlighted that it is a question that remains unanswered. The ongoing study aims to contribute to the literature on how the components of expectancy, value, and cost influence student motivation in the context of high school mathematics.

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